PRESSURE DISTRIBUTION BETWEEN PARALLEL PLATES ARISING FROM THE MOLECULAR VISCOUS FLOW OF VAPOR DURING THE SUBLIMATION OF ICE

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Theoretical and experimental relationships are derived for the pressure distribution between two parallel subliming plates. The possibility of using the analytical relationships so obtained in order to choose the optimum slit height in industrial sublimation plants is discussed.

Many papers have been written on the determination of optimum procedures for the sublimational drying of food and biological products with various forms of power supply. Chief attention has been paid to intensifying the drying process while preserving the natural taste and nutritional properties of the product. It should be noted that, in certain cases (drying on shelves with radiative—conductive heat supply, drying in ampoules, etc.), increasing the efficiency of the apparatus is mainly associated with reducing the distance between the heat- and mass-transfer surfaces. However it has never been made clear to what extent this distance may in fact be reduced.

If sublimation occurs in narrow, slot-like channels, a certain temperature and pressure drop is set up along the channel; it depends on the intensity of sublimation and the total pressure in the chamber, and also on the geometrical dimensions of the slot. A rise in pressure in the center of the slot may lead to local overheating of the surface of the material, and even to a technological breakdown of the drying process. For practical applications it is therefore essential to establish fairly reliable rules as to the choice of the best distance between the walls of the slot.

The hydrodynamics of the flow of a vapor in narrow gaps between circular subliming disks under viscous conditions ($Kn < 0.01$) were studied in reference 1.

We subsequently attempted a theoretical and experimental investigation into the vapor-pressure distribution of the sublimate in slots of rectangular configuration for the molecular-viscous type of flow ($0.01 < Kn < 0.1$).

Let us consider the three-dimensional flow of a rarefied vapor in a slot channel (Fig. 1). We assume that the slot channel has a medium surface and its height 2h satisfies the condition $(\nabla h) \ll 1$ where $\nabla = \bar{i}\partial/\partial x + \bar{j}\partial/\partial y$. In the symmetry plane we use the rectangular coordinates 0xy. The distances from this plane are measured by the coordinate z. Since the height of the slot channel is small compared with the scale of the flow vapor in the 0xy plane, the equations of motion, continuity, and heat conduction may be expressed in the following form (as in the analysis of flows of incompressible liquid in a narrow gap [2], we specially separate out the transverse velocity component $\bar{v} = \bar{u} + \bar{k}w$):

$$\mu \frac{\partial^2 \bar{u}}{\partial z^2} - \nabla P = \rho (\bar{u}\nabla) \bar{u} + \rho w \frac{\partial u}{\partial z} - \mu \nabla^2 \bar{u}, \tag{1}$$

$$\frac{\partial P}{\partial z} = \mu \frac{\partial^2 w}{\partial z^2} - \rho w \frac{\partial w}{\partial z} - \rho \bar{u} \nabla w + \mu \nabla^2 w, \tag{2}$$

$$\frac{\partial (\rho w)}{\partial z} + \nabla (\rho \bar{u}) = 0, \tag{3}$$

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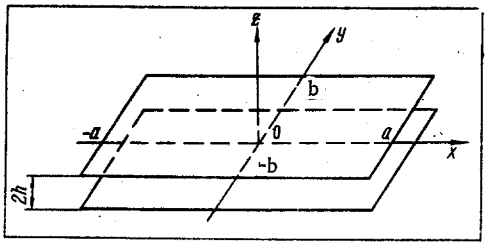

Fig. 1. Scheme of slot.

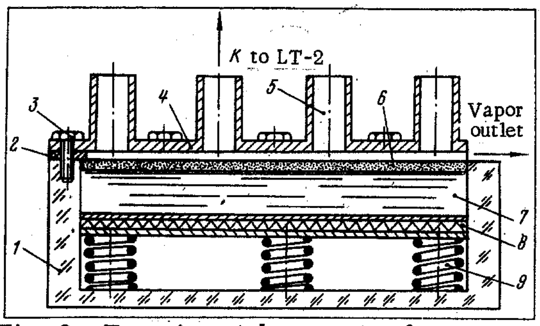

Fig. 2. Experimental apparatus for measuring vapor pressure in a plane slot.

$$\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)=c_p\rho\bar{u}\nabla T+c_p\rho w\frac{\partial T}{\partial z}-\nabla(\lambda\nabla T). \quad (4)$$

At the walls of the channel [$z = \pm h(x, y)$] conditions corresponding to the occurrence of slip and a temperature jump have to be satisfied:

$$u=\frac{2-\sigma}{\sigma}l\left(\frac{\partial u}{\partial z}\right)_{z=-h}+\frac{3}{4}\frac{\mu}{\rho T}\nabla T, \quad (5)$$

$$T=T_w+\frac{2-\alpha}{\alpha}\frac{2\gamma}{\gamma+1}\frac{l}{\mathrm{Pr}}\left(\frac{\partial T}{\partial z}\right)_{z=-h}. \quad (6)$$

Let us consider the laminar flow of vapor in a slot channel, corresponding to the small Reynolds numbers which characterize these, flows in the same way as flows of incompressible liquid of the Hill—Shaw type.

The vapor is considered as an ideal gas, i.e., we use the equation of state

$$\rho=P/RT. \quad (7)$$

Using Eqs. (1)-(3), we may obtain an estimate for the ratio of the pressure drop over the height of the slot to the pressure drop in the symmetry plane $\Delta P_h/\Delta P_L = 0(h^2/L^2)$. If $h^2/L^2 \ll \mathrm{Re}$, and remembering that for gases $\mathrm{Pr} \lesssim 1$, we may use Eqs. (4) and (6) to derive the following estimate for the ratio of the temperature drop over the height of the slot ΔT_h to the temperature drop along the symmetry plane ΔT_L: $\Delta T_h/\Delta T_L = 0(\mathrm{Pe}) = 0(\mathrm{Re})$. We may thus consider that over the height of the slot the vapor parameters P, T, ρ are practically constant, and that sublimation at the walls takes place in a quasi-equilibrium manner, i.e., the temperature and pressure of the vapor in the slot are related by the Clapeyron—Clausius equation

$$T=\frac{\Lambda}{R}F(P),\quad F(P)=\frac{RT_*/\Lambda}{1-(RT_*/\Lambda)\ln P/P_*}.$$

Since the ratio of the terms on the right-hand side of Eq. (1) to the terms on the left-hand side are quantities of the order of Re or h^2/L^2, in the linear approximation (to which we shall here confine attention) they may be neglected, and we may write the following expression for the distribution of the longitudinal component of the velocity of the vapor over the height of the slot:

$$\bar{u}=-\frac{h^2}{2\mu}\left(1+2\frac{2-\sigma}{\sigma}\frac{l}{h}-\frac{z^2}{h^2}\right)\nabla P+\frac{3}{4}\frac{\mu}{\rho T}\nabla T.$$

Remembering that

$$l=1.26\mu\sqrt{\gamma}/\rho a,\quad a=\sqrt{\gamma RT},$$

we obtain

$$\bar{v}=\frac{1}{h}\int_0^h\bar{u}dz=-\frac{h^2}{3\mu}\left[1+\frac{2-\sigma}{\sigma}\frac{3.78\mu}{\rho h\sqrt{RT}}-\frac{9}{4}\frac{\mu^2}{\rho h^2 T}\frac{d\ln F(P)}{dP}\right]\nabla P.$$

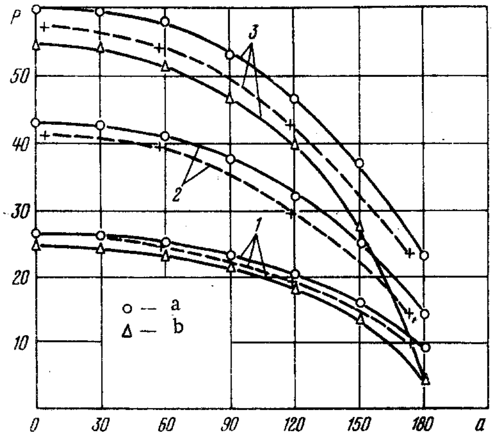

Fig. 3. Vapor pressure P (N/m^2) along the flat tube as a function of distance a (mm): 1) 2h = 4 mm and $J_m = 1.8 \cdot 10^{-4}$ kg/m^2·sec; 2) 4 and $4 \cdot 10^{-4}$; 3) 4 and $7.7 \cdot 10^{-4}$. Continuous curves, theoretical data; broken curves, experiment.

We find the pressure distribution in the gap between the subliming surfaces from the material-balance equation

$$\nabla (h\rho v) = J_m. \tag{8}$$

If the height of the slot is constant, the latter equation may be expressed in the form

$$\nabla^2 \Psi = -\frac{3\mu J_m}{h^3}, \quad \Psi = \int_{P_*}^{P} (\Phi_1 + \Phi_2 + \Phi_3)\, dP,$$

where

$$\Phi_1 = \frac{P}{RT}, \quad \Phi_2 = \frac{2-\sigma}{\sigma} \frac{3.78\mu}{h\sqrt{RT}}, \quad \Phi_3 = -\frac{9}{4}\frac{\mu^2}{h^2}\frac{d \ln F(P)}{dP}, \tag{9}$$

$$\Psi_1 = \frac{P^2 - P_*^2}{2RT_*} - \frac{1}{\Lambda}\frac{P^2}{2}\ln\frac{P}{P_*} + \frac{P^2 - P_*^2}{4\Lambda},$$

$$\Psi_3 = \frac{9}{4}\frac{\mu^2}{h^2}\ln\left(1 - \frac{RT_*}{\Lambda}\ln\frac{P}{P_*}\right). \tag{10}$$

If $\left|\frac{RT_*}{\Lambda}\ln\frac{P}{P_*}\right| \ll 1$, then in the expression

$$\sqrt{1 - \frac{RT_*}{\Lambda}\ln\frac{P}{P_*}} = 1 - \frac{1}{2}\frac{RT_*}{\Lambda}\ln\frac{P}{P_*} + \ldots$$

we may confine attention to the first few terms. Then

$$\Psi_2 = \frac{2-\sigma}{\sigma}\frac{3.78\mu}{h\sqrt{RT_*}}\left[P - P_* - \frac{1}{2}\frac{RT_*}{\Lambda}P\left(\ln\frac{P}{P_*} + \frac{P_*}{P} - 1\right) - \frac{1}{8}\left(\frac{RT_*}{\Lambda}\right)^2 P\left(\ln^2\frac{P}{P_*} - 2\ln\frac{P}{P_*} - 2\frac{P_*}{P} + 2\ldots\right)\right]. \tag{11}$$

If $J_m = J_m(x, y)$, the determination of the pressures in the slot reduces to finding the solution Ψ to the boundary problem for the Poisson equation (8). The problem is greatly simplified if we consider that J_m = const along the length of the slot.

As a specific example, let us consider the flow of vapor in the narrow gap between subliming parallel plates of finite length and width $|x| < a$, $|y| < b$ (Fig. 1). The external pressure is assumed equal to P', i.e., for $x = \pm a$ or $y = \pm b$ $\Psi = \Psi' = \Psi(P')$. Then for a uniform distribution of heat sources along the surface (J_m = const) we we shall have

$$\Psi = \Psi' + \frac{3\mu J_m}{2h^3}(a^2 - x^2) + \frac{48\mu J_m a^2}{\pi^3 h^3}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(2n+1)^3}\frac{\cos(\mu_n x)\,\mathrm{ch}(\mu_n y)}{\mathrm{ch}(\mu_n b)}, \tag{12}$$

where $\mu_n a = \pi(n + 1/2)$.

The maximum value of the vapor pressure occurs at the point x = 0, y = 0, where for $b \geq a$

$$\Psi = \Psi' + \frac{3\mu J_m a^2}{2h^3}\left[1 - \frac{32}{\pi^3}\frac{1}{\operatorname{ch}\left(\frac{\pi}{2}\frac{b}{a}\right)}\right].$$

Being in possession of the relationships $\Psi = \Psi(P)$ (9)-(11) and $\Psi = \Psi(x)$ (12), we may determine the pressure at any point of the slot.

In order to verify the foregoing analytical relationships (9)-(12), we set up a special experiment to measure the pressures in the slot. The arrangement of the experimental apparatus, with slot dimensions of 180 × 60 mm, is illustrated in Fig. 2. A plate of ice 7 was placed in a rectangular dish 1. On the slot side the ice was covered with a thin plate of porous titanium 6, which simulated the plane sublimation surface. Heat was supplied conductively from the electric heater 8, which had the same dimensions as the ice plate. The connection between the lid 4 and the dish 1 was made airtight by using the bolts 3 to compress the rubber gasket 2; the gasket lay around the whole perimeter except for one of the sides, through which the vapor was carried away during sublimation. The heater, ice, and porous plate were pressed against the support with the rubber gasket 2 by means of the springs 9; this enabled us to maintain a constant height of the slot during the whole period of the experiment. The height of the slot was taken as 0.8, 2, or 4 mm; the actual value was determined by an appropriate choice of spacers (gaskets). In this way we achieved the simplest case: the flow of vapor in the flat tube when one of the ends of the tube was closed. The last term in Eq. (12) therefore vanished, since for the present case in effect $b = \infty$.

The pressure along the slot was measured by means of a set of thermocouple manometers (LT-2 tubes), which were hermetically connected to the pipes 5. The tubes LT-2 were placed in the vacuum chamber at as short a distance as possible from the sublimation surface (150-200 mm). All four tubes were connected to a single VT-3 vacuum gage through a system of switches. This enabled us to measure the pressures in the slot very precisely. Thus, for example, when the pressure in the chamber equalled ~133 N/m^2 the scatter in the readings of the tubes measuring this pressure was no greater than ±5%. The error in measuring the pressures with the VT-3 vacuum gage was ±15%. Thus the pressure in the slot was measured with a total error not exceeding ±20%. In determining the pressure in the slot we also introduce a corresponding correction allowing for the difference between the thermal conductivities of the rarefied water vapor and air.

The experiments were carried out with a pressure of ~4 N/m^2 in the chamber.

The results of our theoretical and experimental determinations for $2h = 4$ mm ($Kn \approx 0.1$) are presented in Fig. 3. Curves 1a, 2a, and 3a were obtained from Eqs. (9)-(12); however, the pressure at the outlet from the slot P' was taken as being close to the vapor pressure measured experimentally with the manometer tubes. In practice the pressure of the surroundings is usually known and the pressures along the slot are therefore calculated on the basis of this value. Figure 3 also shows the results of some calculations (curves 1b and 3b) for which the general pressure in the chamber was taken as P'. For small values of J_m, the theory agrees satisfactorily with experiment along the whole slot, except for the peripheral regions.

The vapor pressure in the slot section $x = 0$ is 2.5-3 times greater than at the cut-off section, while the pressure drop along the vapor flow increases with increasing J_m. It should be noted that the vapor pressure at the outlet from the slot may be several times as great as that in the vacuum chamber. Hence the pressure of the escaping vapor relaxes to the general pressure in the chamber at a certain distance from the slot cut-off.

For $2h = 0.8$ and 2 mm ($Kn \approx 0.9$ and 0.35, respectively) and the same values of J_m the pressure in the slot calculated from Eqs. (9)-(12) is 1.5-2 times greater than the experimental value. This indicates that for the transient and molecular modes of vapor flow in narrow slots other relationships are required, and the establishment of these lies outside the scope of the present article.

Thus for the sublimation process and the flow of vapor in a narrow slot between parallel plates at Knudsen numbers of under 0.1 we have here derived some simple analytical relationships enabling us to determine the vapor pressure of the sublimate along the slot to an accuracy sufficient for all practical purposes. Final choice of optimum slot height in industrial sublimation systems of the tray type must be made separately in each specific case by seeking those conditions which establish the maximum possible pressure and hence the maximum temperature of the surface of the material inside the slot.

NOTATION

J_m	is the sublimation intensity;
$2h$, $2a$, and $2b$	are the height, length, and width of the slot;
T, P, ρ, μ	are the temperature, pressure, density, and dynamic viscosity of the vapor;
T_* and P_*	are the temperature and pressure of the vapor on the line of saturation;
$\gamma = c_p/c_v$;	
R	is the gas constant;
Λ	is the latent heat of sublimation;
σ	is the proportion of molecules reflected diffusely from the walls ($\sigma = 0.9$);
v	is the integrated mean velocity of the vapor;
L	is the characteristic dimension in the plane of the slot.

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